Maths Basic
CBSE Class-X (2025)

Series-GE1FH	QP Code-430/1/1	Set-I

Section-A

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each. Internal choice is provided in 2 marks questions in each case study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Draw neat diagrams wherever required. Take = wherever required, if not stated.

(x) Use of calculator is not allowed.

SECTION A

This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each. 20 1=20

1. *If the HCF of two positive integers a and b is 1, then their LCM is:*

   (A) $a + b$ (B) a

   (C) b (D) ab [1]

Solution: (D) ab

If the Highest Common Factor (HCF) of two positive integers a and b is 1, it means they are co-prime i.e. they have no common factors other than 1.

HCF × LCM = a × b

Or, 1 x LCM = a x b

Or, LCM = ab (Ans.)

2. The number $3 + \sqrt{2}$ is:

(A) a rational number *(B) an irrational number*

(C) an integer *(D) a natural number*

Solution: (B) an irrational number

Explanation:

Let`s analyze, each component of $3 + \sqrt{2}$,

3 is a rational number (can be expressed as $\frac{3}{1}$).

whereas $\sqrt{2}$ is an irrational number (it cannot be expressed as a fraction and its decimal expansion is non-terminating and non-repeating).

Adding a rational number (3) and an irrational number ($\sqrt{2}$) always gives an irrational number.

3. The discriminant of the quadratic equation $x^2 - 3x - 2 = 0$ is :

(A) 1 (B) 17

(C) $\sqrt{17}$ (D) $-\sqrt{17}$

Solution: (B) 17

Given,

$$x^2-3x-2=0$$

The discriminant (D) of a quadratic equation in the form $ax^2+bx+c = 0$ is:

$D = b^2-4ac$

Substituting the value of a = 1, b = -3 and c = -2 we get,

$D=(-3)^2-4\,(1)\,(-2) = 9 + 8 = 17$

4. The equation $x + \frac{1}{x} = 3\ (x \neq 0)$ is expressed as a quadratic equation in the form of $ax^2 + bx + c = 0$. The value of a - b + c is :

(A) 5 *(B) 2*

(C) 1 *(D) 1*

Solution: (A) 5

Explanation:

Given,

$$x + \frac{1}{x} = 3$$

$x^2 + 1 = 3x$ (After multiplying by x on both the sides)

or, $x^2 - 3x + 1 = 0$

Now, compare with the quadratic equation $ax^2 + bx + c = 0$

We have,

a = 1

b = - 3

c = 1

Now, a – b + c = 1 – (-3) + 1 = 1 + 3 + 1 = 5

Value of a – b + c = 5.

5. For a point (3, - 5), the value of (abscissa - ordinate) is:

(A) - 8 *(B) - 2*

(C) 2 *(D) 8*

Solution: (D) 8

Explanation:

Given point (3, -5)

Abscissa (x-coordinate) = 3

Ordinate (y-coordinate) = – 5

Now, abscissa – ordinate = 3 – (-5) = 3 + 5 = 8 (Ans.)

6. The mid-point of a line segment divides the line segment in the ratio:

(A) 1 : 2 *(B) 2 : 1*

(C) 1 : 1 *(D)* $\frac{1}{2}$*: 2*

Solution: (C) 1: 1

Explanation:

The mid-point of a line segment is the point that divides the segment exactly in half.

So, it divides the line segment in the ratio:

1:1

7. Which of the following is not the criterion for similarity of triangles?

(A) AAA *(B) SSS*

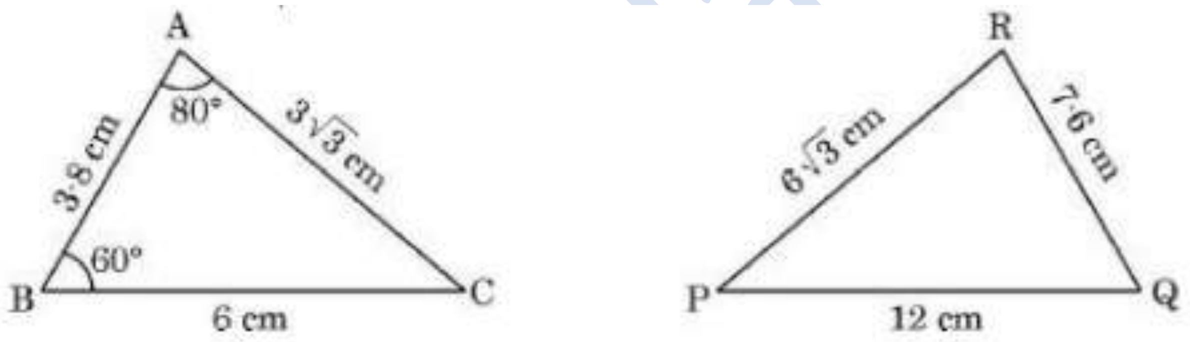
(C) SAS *(D) RHS*

Solution: (D) RHS

Explanation:

the similarity criteria for triangles include SAS (Side-Angle-Side), SSS (Side-Side-Side) and AAA (Angle-Angle-Angle). RHS (Right angle-Hypotenuse-Side) is not a criterion for similarity but for congruence.

8. From the figures given below, which of the following is true about the measure of ∠P ?



(A) ∠P = 60°

(B) ∠P = 80°

(C) ∠P = 40°

(D) The measure of ∠P cannot be determined

Solution: (C) ∠P = 40°

Explanation:

In △ ABC,

AB = 3.8 cm,

BC = 6 cm

CA = 3√3 cm

∠BAC = 80°

$\angle ABC = 60°$

So, $\angle ABC + \angle BAC + \angle ACB = 180°$ ***(Sum of all three angles of a △ is 180°)***

$60° + 80° + \angle ACB = 180°$

$\angle ACB = 180 – 140 = 40°$

In △ PQR,

RQ = 7.6 cm

PQ = 12 cm

PR = $6\sqrt{3}$ cm

Now,

$$\frac{AB}{RQ} = \frac{3.8}{7.8} = \frac{1}{2}$$

$$\frac{BC}{PQ} = \frac{6}{12} = \frac{1}{2}$$

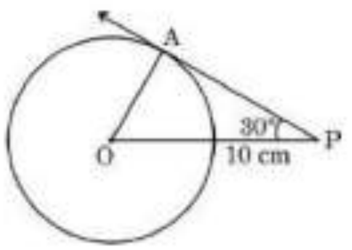
$$\frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

So, $$\frac{AB}{RQ} = \frac{BC}{PQ} = \frac{CA}{PR} = \frac{1}{2}$$

Since both triangles have their corresponding sides are in same proportion. Hence, △ ABC and △ PQR are similar.

therefore, $\angle ACB = \angle P = 40°$ ***(Corresponding angles of similar triangles are equal.)***

9. In the given figure, PA is a tangent to a circle with centre O. If OP = 10 cm, then the length of AP is :



(A) $10\sqrt{3}$ cm

(B) 20 cm

(C) 5 cm

(D) $5\sqrt{3}$ cm

Solution: (D) $5\sqrt{3}$

Explanation:

In the given figure,

OA is perpendicular to the tangent PA because the radius drawn to the point of tangency is perpendicular to the tangent line.

In right angled △ OAP,

$\angle A = 90°$

$\angle P = 30°$

OP = 10 cm (Hypotenuse)

Now, $\text{Cos } 30° = \frac{Adjacent\ side}{Hypotenuse}$

Or, $\frac{\sqrt{3}}{2} = \frac{AP}{OP}$

Or, $\frac{\sqrt{3}}{2} = \frac{AP}{10}$

Or, $AP = \frac{10 \times \sqrt{3}}{2} = 5\sqrt{3}$ cm

10. Which of the following statements is false ?

(A) tan 45° = cot 45°

(B) sin 90° = tan 45°

(C) sin 30° = cos 30°

(D) sin 45° = cos 45°

Solution: (C) sin 30° = cos 30°

Explanation:

$$\sin 30° = \frac{1}{2} \quad \& \quad \cos 30° = \frac{\sqrt{3}}{2}$$

Hence, the statement sin 30° = cos 30° is false.

11. The value of $\left(\tan^2 A - \frac{1}{\cos^2 A}\right)$ is :

(A) more than 1

(B) 1

(C) 0

(D) -1

Solution: (D) -1

Explanation:

Tan^2A - $\frac{1}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A}$ - $\frac{1}{\cos^2 A}$

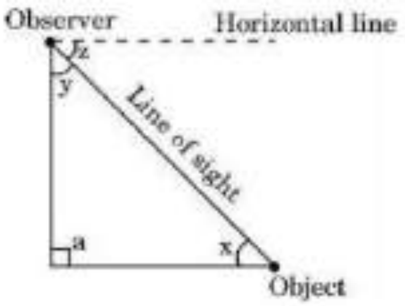
$= \frac{\sin^2 A - 1}{\cos^2 A}$

$= \frac{(1-\cos^2 A)-1}{\cos^2 A}$ (since $\sin^2 A = 1 - \cos^2 A$)

$= \frac{-\cos^2 A}{\cos^2 A}$

= -1

12. In the given figure, which of the following angles represents the angle of depression ?



(A) x
(B) y
(C) z
(D) a

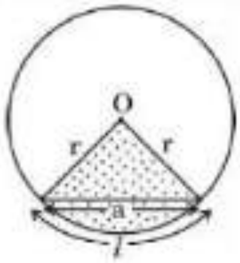
Solution: (C) z

Explanation:

In the given figure, the angle of depression is the angle between the horizontal line (from the observer's eye level) and the line of sight to the object.

Hence, z forms the angle of depression

13. The perimeter of the shaded region in the given figure is :



(A) l

(B) $l + a$

(C) $l + 2r$

(D) $l + 2r + a$

Solution: (C) l + 2r

Explanation:

In the given figure,

The shaded part includes the two radius each of length r and the arch length (l) of the circle. The base a will not be included as this is not part of the outer boundary of the shaded region.

Hence, the perimeter of the shaded region = l + r + r = l + 2r

14. The ratio of the area of a quadrant of a circle to the area of the same circle is :

(A) 1 : 2 *(B) 2 : 1*

(C) 1 : 4 *(D) 4 : 1*

Solution: (C) 1:4

Explanation:

A quadrant is one-fourth of a full circle.

Lets the radius of the circle is r.

Area of the circle = πr^2

Area of a quadrant of the circle = $\frac{1}{4}\pi r^2$

$$\frac{Area\ of\ a\ quadrant\ of\ a\ circle}{Area\ of\ the\ circle} = \frac{\frac{1}{4}\pi r2}{\pi r2} = \frac{1}{4}$$

15. For which of the following solids is the lateral / curved surface area and total surface area the same?

(A) Cube *(B) Cuboid*

(C) Hemisphere *(D) Sphere*

Solution: (D) Sphere

Explanation:

Lateral surface area of a sphere = $4 \pi r^2$.

Total surface area of the sphere = $4 \pi r^2$

Hence, the lateral surface area and the total surface area of a sphere is same.

16. The class mark of the median class of the following data is :

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

(A) 40 ***(B) 55***

(C) 47·5 ***(D) 62·5***

Solution: (D) 62.5

Explanation:

First of all, find the cumulative frequency of each class interval as shown below:

Class Interval	**10-25**	**25-40**	**40-55**	**55-70**	**70-85**	**85-100**
Frequency	2	3	7	6	6	6
Cumulative Frequency	2	5	12	18	24	30

N/2 = sum of all frequencies/2 = 30/2 = 15

Since 15 lies in between the cumulative frequency 12 and 18, so we take the median class 55-70.

Now, class mark of the median class = $\frac{Lower\ limit\ of\ median\ class + upper\ limit\ of\ median\ class}{2}$

$= \frac{55+70}{2}$

$= \frac{125}{2} = 62.5$

17. The following distribution shows the number of runs scored by some batsmen in test matches :

Runs scored	*3000-4000*	*4000-5000*	*5000-6000*	*6000-7000*
Number of batsmen	*5*	*10*	*9*	*8*

The lower limit of the modal class is:

(A) 3000 *(B) 4000*

(C) 5000 *(D) 6000*

Solution: (B) 4000

Explanation:

We know that modal class is the class interval with highest frequency. In the given distribution of runs, the highest frequency is 10 which corresponds to the class interval 4000 – 5000.

Hence, lower limit of the class interval (4000 – 5000) is 4000.

18. In a random experiment of throwing a die, which of the following is a sure event?

(A) Getting a number between 1 and 6

(B) Getting an odd number < 7

(C) Getting an even number < 7

(D) Getting a natural number < 7

Solution: (D) Getting a natural number < 7.

Explanation:

The possible outcomes when a die is thrown are:

(1,2,3,4,5,6)

Let's consider each option one by one:

- Option (A): Getting a number between 1 and 6. This means that the possible outcome number will be 2, 3, 4 & 5. It excludes 1 and 6, so it does not cover all outcomes.
- Option (B): Getting an odd number < 7.
  This includes the possible outcome number (1, 3, 5) and missing numbers like 2, 4, 6. So, this is not a sure event.
- Option (C): Getting an even number < 7.
  This includes the possible outcome number (2, 4, 6) and missing numbers like 1, 3, 5. Again, not a sure event.
- Option (D): Getting a natural number < 7.
  Natural numbers less than 7 are (1, 2, 3, 4, 5, 6) which exactly matches all the possible outcomes of a die.. Hence, this is a sure event.

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : For any two natural numbers a and b, the HCF of a and b is a factor of the LCM of a and b.

Reason (R) : HCF of any two natural numbers divides both the numbers.

Solution: (A)

Explanation:

Assertion: The HCF (Highest Common Factor) of any two natural numbers is always a factor of their LCM (Least Common Multiple). This is the fundamental property of HCF.

Reason: The HCF of any two natural numbers divides both numbers. This is the definition of HCF.

Why R explains A: Since the HCF of two numbers divides both numbers, it must be a factor of any common multiple of those numbers. The LCM is the smallest common multiple, so the HCF must be a factor of the LCM.

20. Assertion (A) : The value of p for which the system of equations 4x + py + 8 = 0 and 2x + 2y + 2 = 0 is consistent is 4.

Reason (R) : The system of equations a1x + b1y = c1 and a2x + b2y = c2 is consistent with infinitely many solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Solution: (D)

Explanation:

We will check if the given system is consistent for $p=4$ and if the condition for infinitely many solutions is met. Now put the value of P in the given equation.

4x + 4y + 8 = 0

Here, $a_1 = 4$, $b_1 = 4$ & $c_1 = 8$

$2x + 2y + 2 = 0$

Here, $a_1 = 2$, $b_1 = 2$ & $c_1 = 2$

Now,

$$\frac{a_1}{a_2} = \frac{4}{2} = \frac{2}{1}$$

Similarly,

$$\frac{b_1}{b_2} = \frac{4}{2} = \frac{2}{1}$$

Similarly,

$$\frac{c_1}{c_2} = \frac{8}{2} = \frac{4}{1}$$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Since the ratios of the co-efficient is not equal, the given equation is not inconsistent. Hence, the assertion (A) is false.

The reason (R) is true because a system of linear equations is consistent if it has at least one solution (unique or infinitely many). For infinitely many solutions, the ratios of corresponding coefficients must be equal.

SECTION B

This section has 5 Very Short Answer (VSA) type questions carrying 2 marks each.

21. Solve the following system of equations for x and y

$$\frac{x}{2} + \frac{2y}{3} = -1 \text{ and } x - \frac{y}{3} = 3$$

Solution:

$\frac{x}{2} + \frac{2y}{3} = -1$ (Given)

$\frac{3x+4y}{6} = -1$

$3x + 4y = -6$

$3x = -6 - 4y$ ……………eq. (i)

Now,

$x - \frac{y}{3} = 3$

$\frac{3x-y}{3} = 3$

3x – y = 9

(-6 - 4y) -y =9 *(putting the value of 3x from eq.(i))*

- 6 – 5y = 9

5y = -15

$Y = \frac{-15}{3} = -3$

After putting the value of y in eq. (i) we get,

3x = -6 – 4 x -3

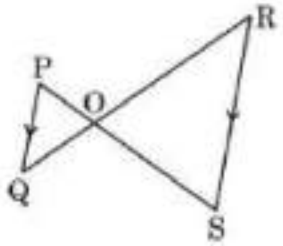
3x = -6 +12

3x = 6

X = 2

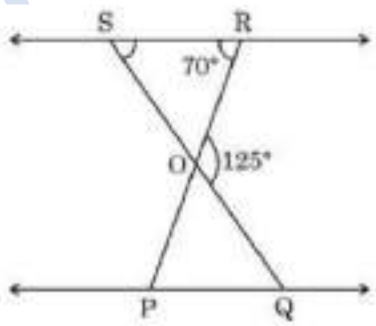
Value of x = 2 and Y = -3

22. (a) In the given figure, if PQ ∥ RS, then prove that △POQ ~△ SOR.



OR

(b) In the given figure, △ OSR ~ △ OQP, ∠ROQ = 125 and ∠ORS = 70 . Find the measures of ∠OSR and ∠OQP.



Solution: 22 (a)

We know that if two parallel lines are intersected by a transversal line, the alternate interior angles formed by the transversal lines are equal.

Here, PQ ∥ RS,

$\angle PQO = \angle ORS$ (Alternate interior angles)

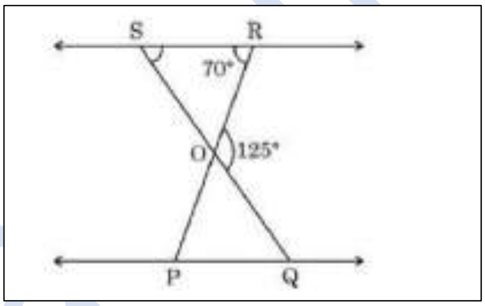
Similarly, $\angle OPQ = \angle OSR$ (Alternate interior angles)

$\angle POQ = SOR$ (vertically opposite angles)

We conclude that,

△POQ ~△ SOR (AAA Criterion)

Solution: 22 (b)



Given,

△ OSR ~ △ OQP

$\angle ROQ = 125°$

$\angle ORS = 70°$,

Now in △ OSR,

$\angle ORS + \angle OSR = \angle ROQ$

(Exterior angle of a △ is equal to the sum of the two non-adjacent interior angles)

$70° + \angle OSR = 125°$

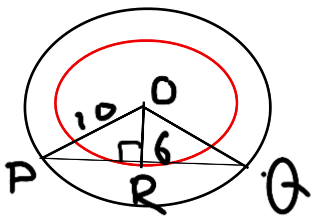
Or, $\angle OSR = 125° - 70° = 55°$

Now,

$\angle OQP = \angle OSR = 55°$ (since △ OSR ~ △ OQP, their corresponding angles are equal).

23. Two concentric circles are of radii 6 cm and 10 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:



From the given fig.

O is the centre of the two concentric circles where OP is the radius of the larger circle and OR is the radius of the smaller circle.

$OP = 10$ cm and $OR = 6$ cm

We know that a radius of a circle is always perpendicular to a tangent line at the point of intersection.

In $\triangle$ OPR,

$\angle ORP = 90°$

$OP^2 = OR^2 + PR^2$

Or, $10^2 = 6^2 + PR^2$

Or, $PR^2 = 100 - 36 = 64$

Or, $PR = 8$ cm

Similarly, RQ = PR = 8 cm

Length of the chord PQ = PR + RQ = 8 + 8 = 16 cm.

24. (a) Find the values of A and B ($0 \leq A < 90°$, $0 \leq B < 90°$), if $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$

or,

24(b) Prove that $\tan 45° = 1$ geometrically.

Solution: 24 (a)

$\tan(A+B) = 1$ (given)

or, $\tan(A+B) = \tan 45°$

or, $A+B = 45°$ ------------ eq.(i)

Similarly, $\tan(A-B) = \frac{1}{\sqrt{3}}$

Or, $\tan(A-B) = \tan 30°$

Or, $A - B = 30°$

Now,

$$\begin{array}{r} A+B = 45° \\ A-B = 30° \\ \hline 2A = 75° \end{array}$$

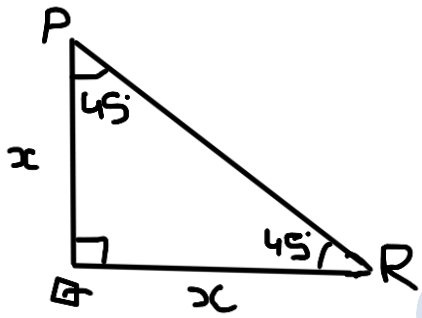
A = 75°/2 = 37.5°

37.5° + B = 45° (putting the value in eq.(i))

B = 45° - 37.5° = 7.5°

Value of A = 37.5° and B = 7.5°

Solution: 24(b)



Constructed an isosceles right angle △ PQR in which,

∠PQR = 90°,

QP = QR= x cm (isosceles triangle)

Now in right angled △ PQR,

∠QPR = ∠PRQ = 45° (isosceles right angle △)

Tan 45° = $\frac{Perpendicular}{Base}$

Or, Tan 45° = $\frac{PQ}{QR}$

Or, Tan 45° = $\frac{x}{x}$

Or, Tan 45° = 1 (proved)

25. A chord of a circle of diameter 20 cm subtends an angle of 60 at the centre of the circle. Find the area of the corresponding minor segment of the circle. (Use π = 3·14 and $\sqrt{3}$= 1·73)

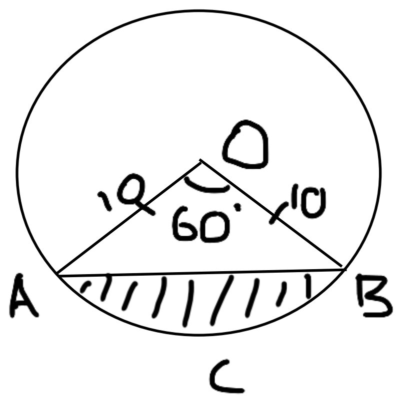
Solution:

From the figure,

O is the centre of the circle.

OA = OB = 10 cm = radius of the circle

∠AOB = 60°



In $\triangle OAB$,

OA=OB (radius of the circle)

$\angle OAB = \angle OBA$ …………(angles formed by the equal sides are equal)

$\angle OAB + \angle AOB + \angle OBA = 180°$

$\angle OAB + 60° + \angle OAB = 180°$

$2\angle OAB = 180 - 60$

$\angle OAB = 60$

$\angle OAB = \angle OBA = \angle AOB = 60°$ (Equilateral triangle)

Area of minor segment ACB = area of sector OACB – area of $\triangle OAB$

$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4} \text{ x side}^2$$

$$= \frac{60}{360} \times 3.14 \text{x} 10^2 - \frac{1.73}{4} \text{ x } 10^2$$

$$= 52.33 - 43.25$$

= 9.08 cm² (Ans.)

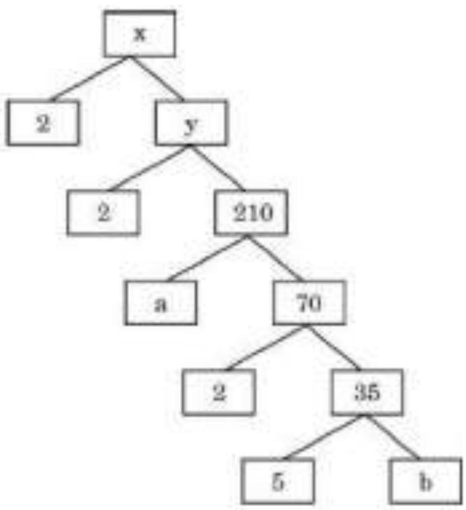
SECTION C

This section has 6 Short Answer (SA) type questions carrying 3 marks each. 6 x3=18

26. (a) Prove that √3 is an irrational number.

OR

(b) The factor tree of a number x is shown below :



Find the values of x, y, a and b. Hence, write the product of the prime factors of the number x so obtained.

Solution: 26 (a)

Let`s assume that $\sqrt{3}$ is a rational number.

This means that $\sqrt{3}$ can be written in the $\frac{P}{q}$ form where p and q is an integer and $q \neq 0$ and p and q is also a co-prime number meaning it does not have common factor other than 1.

Now, $\sqrt{3} = \frac{P}{q}$

Or, $p = \sqrt{3}\,q$

Or, $p^2 = (\sqrt{3}\,q)^2$ (squaring both the sides)

Or, $p^2 = 3q^2$ eq.(i)

Here, 3 is a factor of p^2, therefore 3 is also a factor of p.

Now, $p = 3k$ (say)

Therefore, $(3k)^2 = 3q^2$ (putting the value in eq.(i))

$9K^2 = 3q^2$

Or, $3K^2 = q^2$

Or, $q^2 = 3K^2$

Here, 3 is a factor of q^2. Therefore, 3 is also a factor of q.

Since, p and q have more than one common factors i.e 1 & 3, it means that p and q is not a co-prime number which contradict the aforesaid statement that p and q is a co-prime number. Therefore, we can conclude that $\sqrt{3}$ is an irrational number.

Solution: 26(b)

From the given factor tree, it is clear that

b x 5 = 35 (given)

$\therefore b = 35/5 = 7$

Similarly,

a x 70 = 210

$\therefore a = 210/70 = 3$

y = 2 x 210 = 420

And, 2 x 420 = x

x = 840

therefore, value of x = 840, y = 420, a = 3 and b = 7

27. Find a quadratic polynomial whose sum and product of zeroes are 0 and 9, respectively. Also, find the zeroes of the polynomial so obtained.

Solution: 27

Let`s assume that α and β are the two zero of the quadratic polynomial.

Therefore, $\alpha + \beta = 0$ and $\alpha \times \beta = 9$

The general form of quadratic polynomial is

$P(x) = x^2 - (\text{sum of zeroes})\cdot x + (\text{product of zeroes})$

$P(x) = x^2 - (0)\cdot x + (9)$ (on substituting the values)

Hence, the required quadratic polynomial is $P(x) = x^2 + 9$

Now, $x^2 + 9 = 0$

or, $x^2 = -9$

or, $x = \sqrt{-9} = \pm 3i$ (imaginary number)

therefore, the value of $\alpha = +3i$ and value of $\beta = -3i$

28. (a) Solve the following system of equations graphically:

$x + 3y = 6;\ 2x - 3y = 12$

OR

(b) x and y are complementary angles such that x : y = 1 : 2. Express the given information as a system of linear equations in two variables and hence solve it.

Solution:

Given, $x + 3y = 6$

Or, $x = 6 - 3y$

x	3	0
y	1	2

If y = 1,

Then, x = 6 – 3x1 = 6-3 = 3

And, if y = 2,

Then, x = 6 – 3 x 2 = 6 – 6 = 0

Now,

$2x - 3y = 12$ (given)

Or, $2x - 12 = 3y$

Or, $y = \frac{2x-12}{3}$

If x = 0,

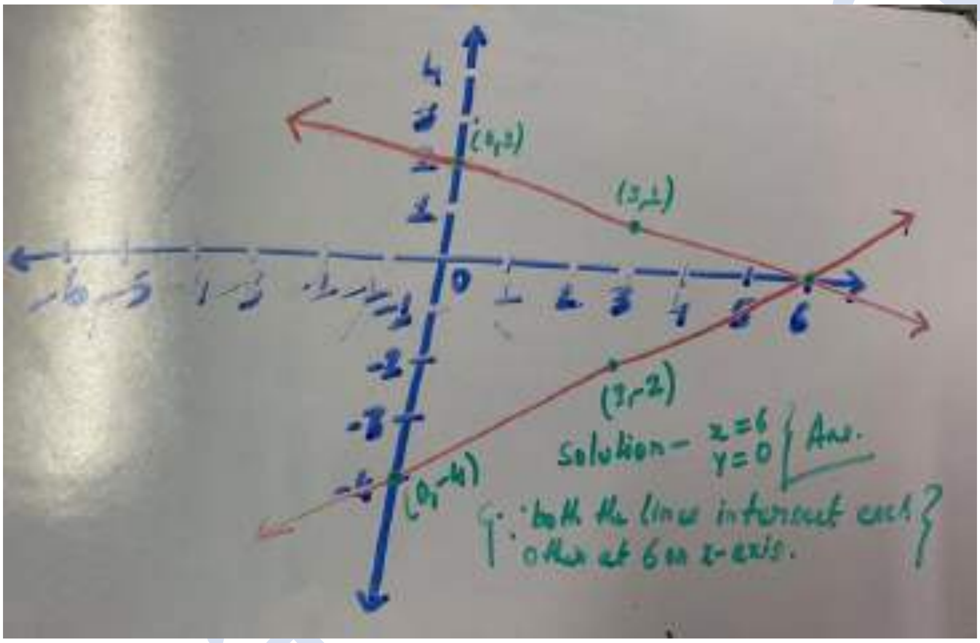
Then $y = \frac{2 \times 0 - 12}{3} = \frac{-12}{3} = -4$

x	0	3
y	-4	-2

If x = 3,

Then $y = \frac{2 \times 3 - 12}{3} = \frac{6 - 12}{3} = -2$

Now, put the values of x and y on the number line. The point at which these two lines intersect each other will be the answer as shown in the picture below.



Solution: 28 (b)

x : y = 1 : 2 (given)

or, $\frac{x}{y} = \frac{1}{2}$

or, $y = 2x$

Now, $x + y = 90°$ (since x and y are complimentary angles, therefore sum of x and y is equal to 90°)

Or, $x + 2x = 90°$

Or, $3x = 90°$

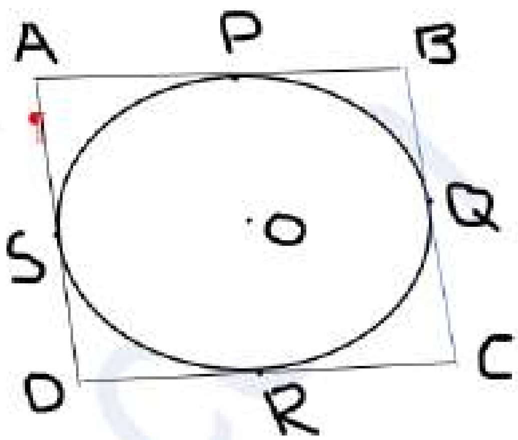
Or, $x = 90/3 = 30°$.

y = 2x = 2 x 30 = 60°

Therefore, value of x = 30° and value of y = 60°.

29. Prove that a rectangle circumscribing a circle is a square.

Solution:



Let`s ABCD is a rectangle in which a circle is circumscribed having the centre O. Point P, Q, R and S is the tangent on the circle.

From the figure,

AP = AS --------------eq.(i) (tangents drawn from an external point are equal)

Similarly, BP = BQ -------------eq (ii)

Similarly, DR = DS ------------eq.(iii)

Similarly, RC = CQ ------------eq.(iv)

Now, adding eq.(i), eq(ii), eq.(iii) & eq.(iv) we get,

AP + BP + DR + RC = AS + BQ + DS + CQ

AB + DC = AD + BC

AB + AB = AD + AD (opposite sides are equal since ABCD is a rectangle)

2AB = 2AD

AB = AD = BC = CD

Hence, ABCD is a square.

30. Prove that:

$$\frac{1+\cot^2 A}{1+\tan^2 A} = \left(\frac{1-\cot A}{1-\tan A}\right)^2$$

Solution:

$$\text{LHS} = \frac{1+\cot^2 A}{1+\tan^2 A}$$

$$= \frac{1+\cot^2 A}{1+\frac{1}{\cot^2 A}}$$

$$= \frac{1+\cot^2 A}{\frac{\cot^2 A+1}{\cot^2 A}}$$

$$= \frac{\cot^2 A(\cot^2 A+1)}{\cot^2 A+1}$$

$$= \cot^2 A$$

$$\text{RHS} = \left(\frac{1-\cot A}{1-\tan A}\right)^2$$

$$= \left(\frac{1-\cot A}{1-\frac{1}{\cot A}}\right)^2$$

$$= \left(\frac{1-\cot A}{\frac{\cot A-1}{\cot A}}\right)^2$$

$$= \left(\frac{\cot A\,(1-\cot A)}{\cot A-1}\right)^2$$

$$= \left(\frac{-\cot A\,(\cot A-1)}{\cot A-1}\right)^2$$

$$= (-\cot A)^2$$

$$= \cot^2 A \qquad = \text{LHS}$$

Hence, LHS = RHS (proved)

31. A lot consists of 200 pens of which 180 are good and the rest are defective. A customer will buy a pen if it is not defective. The shopkeeper draws a pen at random and gives it to the customer. What is the probability that the customer will not buy it ? Another lot of 100 pens containing 80 good pens is mixed with the previous lot of 200 pens. The shopkeeper now draws one pen at random from the entire lot and gives it to the customer. What is the probability that the customer will buy the pen?

Solution:

Total no. of pens = 200

Total no. of good pens = 180

Total no. of defective pens = 200 – 180 = 20

Probability that the customer will not buy the pens = $\frac{Total\ no.of\ defective\ pens}{Total\ no.of\ pens}$

$= \frac{20}{200}$

= 1/ 10

After mixing the two lots of pens,

Total no. of pens = 300

Total no. of good pens = 180 + 80 = 260

Probability that the customer will buy the pen = $\frac{Total\ no.of\ good\ pens}{Total\ no.of\ pens}$

$= \frac{260}{300}$

= 13/15

SECTION D

This section has 4 Long Answer (LA) type questions carrying 5 marks each. *4 x 5=20*

32. (a) The difference of the squares of two positive numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

OR

(b) Find the value(s) of k for which the equation $2x^2 + kx + 3 = 0$ has real and equal roots. Hence, find the roots of the equations so obtained.

Solution: 32 (a)

Let the greater number be x and the smaller number by y.

According to first condition of the question,

$x^2 – y^2 = 180$ -------------- eq.(i)

According to second condition of the question,

$y^2 = 8x$ ---------------eq.(ii)

Now putting the value of y^2 in eq.(i) we get,

$x^2 – 8x = 180$

or, $x^2 – 8x - 180 = 0$

or, $x^2 - 18x + 10x - 180 = 0$

or, $x(x-18) + 10(x-18) = 0$

or, $(x+10)(x-18) = 0$

or, $x + 10 = 0$

or, $x = -10$ (which is not possible)

or, $x - 18 = 0$

or, $x = 18$

putting the value of x in eq.(ii) we get,

or, $y^2 = 8 \times 18 = 144$

or, $y = \sqrt{144} = 12$

Hence, the greater no. = 18 and smaller no. = 12

Or,

Solution: 32(b)

Quadratic equation $2x^2 + kx + 3 = 0$ *(given)*

Comparing the given quadratic equation with the equation $ax^2+bx+c=0$*, we get*

$a = 2$, $b = k$ and $c = 3$

For real and equal roots, the determinants of the equation must be zero

$D = b^2 - 4ac = 0$

Or, $k^2 - 4 \times 2 \times 3 = 0$

Or, $k^2 = 24$

Or, $k = \pm\sqrt{24} = \pm 2\sqrt{6}$

The values of k is $\pm 2\sqrt{6}$

For finding the roots for each k, when roots are equal:

$$x = \frac{-b}{2a}$$

Here $b = k = \pm 2\sqrt{6}$, $a = 2$,

For $k = 2\sqrt{6}$,

$$x = \frac{-2\sqrt{6}}{2.2}$$

$$x = \frac{-2\sqrt{6}}{4}$$

$x = \frac{-\sqrt{6}}{2}$

For, k = - 2√6

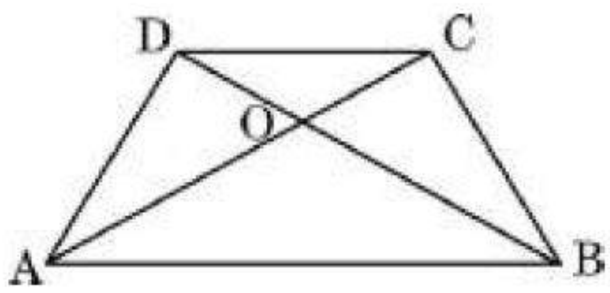
$x = \frac{-(-2\sqrt{6})}{2.2}$

$x = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$

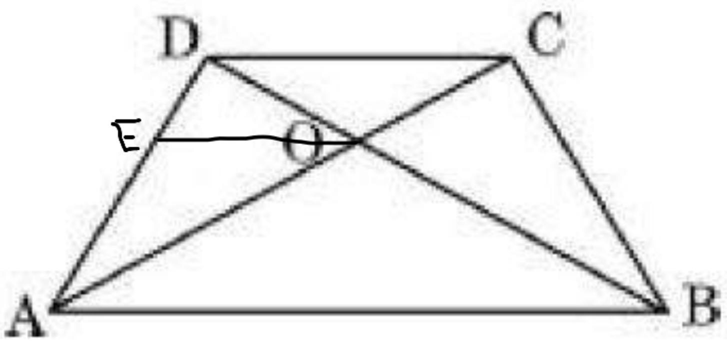
Corresponding roots = ± $\frac{\sqrt{6}}{2}$

33. *State "Basic Proportionality Theorem" and use it to prove the following:*

In a quadrilateral ABCD, diagonals AC and BD intersect each other at O such that $\frac{AO}{BO} = \frac{CO}{DO}$ as shown in the figure. Prove that ABCD is a trapezium.



Solution:



Draw a line EO through point O such that EO is parallel to AB.

In Δ DAB,

Since EO || AB, therefore $\frac{DE}{EA} = \frac{DO}{OB}$ -------------------eq.(i)

(Basic proportionality theorem--if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally)

We are given that $\frac{AO}{BO} = \frac{CO}{DO}$

Or, $\frac{DO}{BO} = \frac{CO}{AO}$ --------------------eq.(ii)

From eq.(i) and eq.(ii) it is clear that,

$\frac{DE}{EA} = \frac{CO}{AO}$

∴ EO || DC *(by the converse of the Basic Proportionality Theorem)*

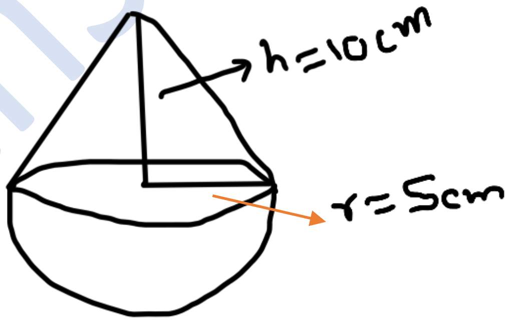
Since EO || AB and EO || DC, then AB || DC. Therefore, the quadrilateral ABCD is a trapezium because one pair of opposite side i.e. AB || DC.

34. (a) A toy is in the form of a cone surmounted on a hemisphere. The cone and hemisphere have the same radii. The height of the conical part of the toy is equal to the diameter of its base. If the radius of the conical part is 5 cm, find the volume of the toy.

OR

(b) A cubical block is surmounted by a hemisphere of radius 3·5 cm. What is the smallest possible length of the edge of the cube so that the hemisphere can totally lie on the cube? Find the total surface area of the solid so formed.

Solution: 34(a)



Given,

Radius of the hemisphere (r) = 5 cm

Height of the cone (h) = diameter = 2r = 2x 5 = 10 cm

Volume of a hemisphere (V_h) = $\frac{1}{2}$ x ($\frac{4}{3}$ πr^3)

= $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi\ (5^3)$

$= \frac{250}{3}\pi\ \ cm^3$

Volume of a cone (V_{cone}) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(5^2)\ (10)$

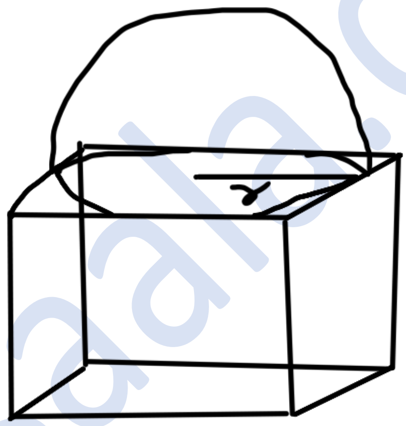
$= \frac{250}{3}\pi\ \ cm^3$

Total volume of the toy = Vol. of hemisphere + Volume of cone

$= \frac{250}{3}\pi\ + \frac{250}{3}\pi$

$\mathbf{= \frac{500}{3}\pi\ \ cm^3}$

Solution: 34 (b)



From the figure,

the radius of the hemisphere (r) = 3.5 cm

Smallest length of edge of the cube = diameter of the hemisphere

$= 2 \text{ x } r = 2 \text{ x } 3.5 = 7$ cm

Total surface area of the solid = total surface area of the cubicle block + curved surface area of the hemisphere – area of the circle

$= 6 \text{ x side}^2 + 2\ \pi r^2 - \pi r^2$

$= 6 \text{ x } 7^2 + \pi r^2$

$= 6 \text{ x } 7 \text{ x } 7 + \frac{22}{7} \text{ x } 3.5^2$

$= 294 + 38.5$

$\mathbf{= 332.5\ cm^2}$

35. The following data gives the information on the observed lifetime (in hours) of 200 electrical components:

Lifetime (in hours)	Number of electrical components
0 - 20	10
20 - 40	35
40 - 60	50
60 - 80	60
80 - 100	30
100 - 120	15

Find the mean lifetime (in hours) of the electrical components.

Solution:

Lifetime (in hours)	Number of electrical components (f)	Midpoint of class interval (x_i)	$f * x_i$
0 - 20	10	10	100
20 - 40	35	30	1050
40 - 60	50	50	2500
60 - 80	60	70	4200
80 - 100	30	90	2700
100 - 120	15	110	1650

$$\text{Mean} = \frac{\sum f * xi}{\sum f}$$

$$= \frac{12200}{200}$$

$= 61$ hours

Mean lifetime of electrical components = 61 hours.

SECTION E

This section has 3 case study based questions carrying 4 marks each. 3*4=12

Case Study-1

36. An injured bird was found on the roof of a building. The building is 15 m high. A fireman was called to rescue the bird. The fireman used an adjustable ladder to reach the roof. He placed the ladder in such a way that the ladder makes an angle of 60 with the ground in order to reach the roof.

Based on the above information, answer the following questions :

(i) Find the length of the ladder used by the fireman to reach the roof.

(ii) Find the distance of the point on the ground at which the ladder was fixed from the bottom of the building.

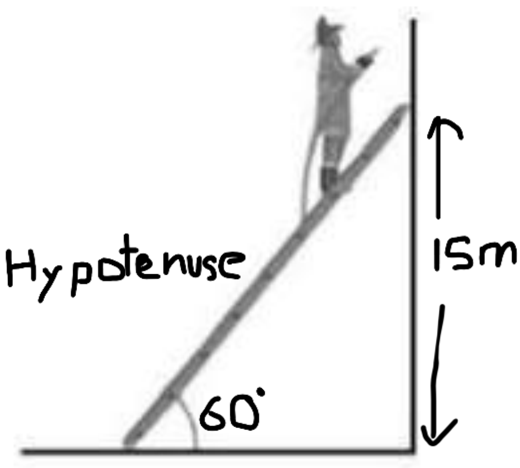
(iii) In order to avoid skidding, the fireman placed the ladder in such a way that the bottom of the ladder touches the base of the wall which is opposite to the building, making an angle of 30 with the ground.

(a) Draw a neat diagram to represent the above situation and hence find the width of the road between the building and the wall.

OR

(b) Find the length of the ladder used by the fireman in this case.

Solution:



Given,

Height of the building = 15 m

Angle formed by the ladder with the ground (θ) = 60°

Now, Sin 60° = $\frac{Perpendicular(P)}{Hypotenuse(H)}$

Or, $\frac{\sqrt{3}}{2} = \frac{15}{H}$

Or, H = $\frac{30}{\sqrt{3}}$ = $10\sqrt{3}$ m.

length of the ladder = $10\sqrt{3}$ m.

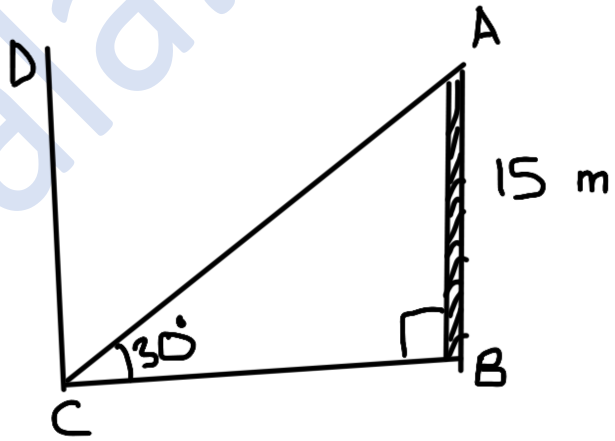
(ii) Now, Cos 60° = $\frac{base(b)}{Hypotenuse(H)}$

Or, $\frac{1}{2} = \frac{b}{10\sqrt{3}}$

Or, b = $\frac{10\sqrt{3}}{2}$ = $5\sqrt{3}$ m.

Distance of the point on the ground at which the ladder was fixed from the bottom of the building = $5\sqrt{3}$ m

36(iii)(a)



Here in the figure,

AB is the height of the building which is 15m and CD is a wall against which the ladder is fixed to avoid skidding. The ladder AC makes an angle 30° with the ground. BC is the width of the road between the building and the wall.

Now, tan 30° = $\frac{Perpendicular}{base}$

Or, $\frac{1}{\sqrt{3}} = \frac{15}{BC}$

Or, BC = $15\sqrt{3}$ m.

Therefore, width of the road between the wall and the building = $15\sqrt{3}$ m

36(iii)(a)

Sin 30° $= \frac{Perpendicular(P)}{Hypotenuse(H)}$

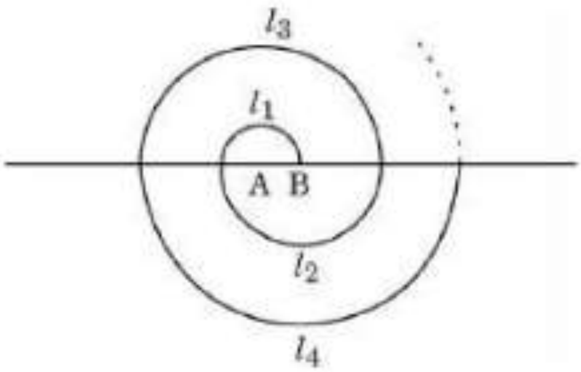
Or, $\frac{1}{2} = \frac{15}{AC}$

Or, AC = 30m

Length of the ladder = 30m.

Case Study 2

37. In a garden, saplings of rose flowers were planted at equal intervals to form a spiral pattern. The spiral is made up of successive semicircles, with centres alternatively at A and B, starting with centre at A, of radii 50 cm, 100 cm, 150 cm, ... as shown in the figure given below. Spiral 1 has 10 flowers, Spiral 2 has 20 flowers, Spiral 3 has 30 flowers and so on.



Based on the above information, answer the following questions:

(i) What is the radius of the 13th spiral? 1

(ii) If the radius of the nth spiral is 500 cm, find the value of n. 1

(iii) (a) Find the total number of saplings till the 11th spiral. 2

OR

(b) Till which spiral, will there be a total of 450 saplings? 2

Solution: 37(i)

Radius of the 1st spiral = 50 cm

Radius of the 2nd spiral = 100 cm

Radius of the 3rd spiral = 150 cm

…………………………………

…………………………………

This pattern of increase shows that the radii of the spiral are in arithmetic progression (AP) with first term $a_1 = 50$ cm and common difference d = 50 cm.

nth term of an AP = $a_1 + (n-1)\ d$

or, $a_n = a_1 + (n-1)\ d$

therefore, $a_{13} = a_1 + (13-1)\ d$ (since we have to find the 13th term)

$= 50 + (13-1)\ 50$

$= 50 + 600$

$= 650$

Solution: 37(ii)

Radius of the nth spiral (a_n)= 500 cm

nth term of an AP = $a_1 + (n-1)\ d$

or, $a_n = a_1 + (n-1)\ d$

or, $500 = 50 + (n-1)\ 50$

or, $(n-1)\ 50 = 500-50$

or, $n-1 = 450/50$

or, $n = 9+1 = 10$

Therefore, on the 10th spiral the radius will be 500cm.

Solution: 37(iii)(a)

Spiral 1 ……….10 flowers,

Spiral 2 ……….20 flowers,

Spiral 3 ………30 flowers

…………………………..

…………………………..

This pattern of increase shows that the spiral are in arithmetic progression (AP) with first term $a_1 = 10$ flowers and common difference d = 10 flowers.

We will have to find the total no. of saplings till 11th spiral.

Now,

Sum of first n terms of an AP = $\frac{n}{2}[2\ a_1 + (n-1)\ d]$

$S_n = \frac{11}{2}[2 \times 10 + (11-1)\ 10]$

$= \frac{11}{2}(120)$

$= 660$ saplings

Total no. of saplings till 11^{th} spiral = 660

or

Solution: 37(iii)(b)

Given,

$a_1 = 10$ flowers; common difference $d = 10$ flowers; $S_n = 450$; $n = ?$

Sum of first n terms of an AP $(S_n) = \frac{n}{2}[2a_1 + (n-1)d]$

0r, $450 = \frac{n}{2}[2 \times 10 + (n-1)10]$

Or, $450 = \frac{n}{2}[10 + 10n]$

Or, $900 = 10n + 10n^2$

Or, $10n^2 + 10n - 900 = 0$

Or, $n^2 + n - 90 = 0$ (on dividing by 10 in each term)

Or, $n^2 + 10n - 9n - 90 = 0$

Or, $n(n+10) - 9(n+10) = 0$

Or, $(n-9)(n+10) = 0$

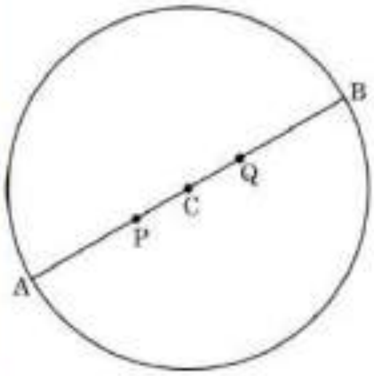
Or, $n-9 = 0$ or, $n+10 = 0$

Or, $n = 9$ or, $n = -10$

Since the value cannot be in negative, therefore on the 9th spiral the total flowers will be 450.

Case Study 3

38. In a society, there is a circular park having two gates. The gates are placed at points A (10, 20) and B (50, 50), as shown in the figure below. Two fountains are installed at points P and Q on AB such that AP = PQ = QB.



Based on the above information, answer the following questions :

(i) Find the coordinates of the centre C. 1

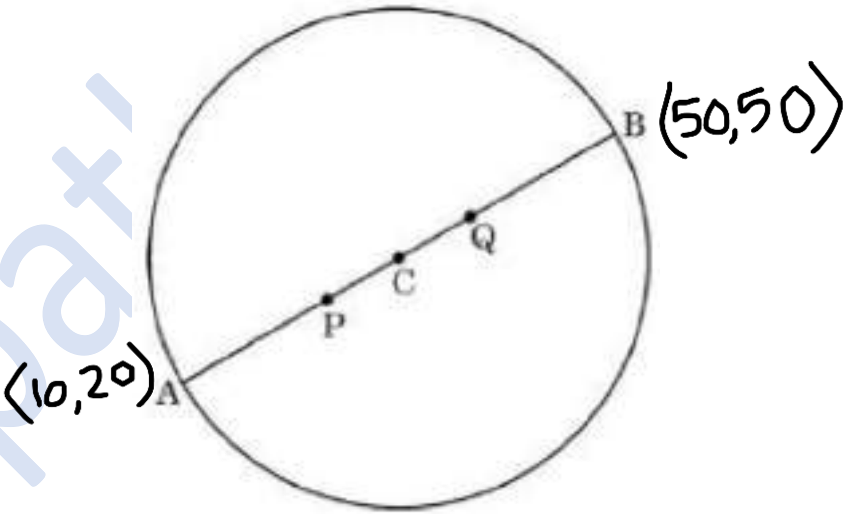
(ii) Find the radius of the circular park. 1

(iii) (a) Find the coordinates of the point P. 2

OR

(b) Find the distance of the fountain at Q from gate A. 2

Solution:



(i) Given,

Co-ordinates of A $(x_1, y_1) = (10, 20)$

Co-ordinates of B $(x_2, y_2) = (50, 50)$

Here, C is the midpoint of AB;

Co-ordinates of C $(x, y) = \left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$ (using midpoint formula)

$= \left(\frac{10+50}{2}, \frac{20+50}{2}\right)$

$= (30, 35)$

(ii) Distance between AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (using distance formula)

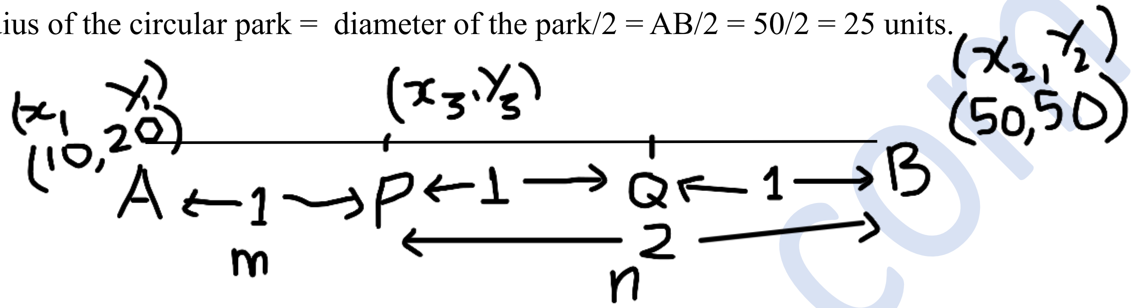
$= \sqrt{(50 - 10)^2 + (50 - 20)^2}$

$= \sqrt{2500}$

$=$ 50 units

Radius of the circular park = diameter of the park/2 = AB/2 = 50/2 = 25 units.

(iii)



In the given figure above,

Since, AP= PQ = QB

Therefore, the ratio of AP to PB = 1 : 2

Now,

$X_3 = \frac{nx1 + mx2}{m+n} = \frac{2x10 + 1x50}{1+2} = \frac{20+50}{3} = \frac{70}{3}$

$Y_3 = \frac{ny1 + my2}{m+n} = \frac{2x20 + 1x50}{1+2} = \frac{40+50}{3} = \frac{90}{3} = 30$

Co-ordinates of P (x_3, y_3) = ($\frac{70}{3}$, 30)

or

(b) In the given figure above,

The line AB is divided into three parts in such a way that AP= PQ = QB

Therefore, $\frac{AQ}{AB} = \frac{2}{3}$

$AQ = \frac{2}{3} \text{ x } AB$

$AQ = \frac{2}{3} \text{ x } 50 = 100/3 = 33.34$ units.

Distance of the fountain at Q from gate A = 33.34 units.

The End